

Drawdown, and the One Excursion That Ends It

Review of Martijn R. Pistorius, *An Excursion-Theoretical Approach to Some Boundary Crossing Problems and the Skorokhod Embedding for Reflected Lévy Processes* (Séminaire de Probabilités XL, Springer LNM 1899, 2007, 21 pp.)

TLDR / 80-20

Stop tracking the path. Count the attempts, then ask what the one fatal attempt did.

Excursion theory splits "when does the drawdown blow through level a " into a rate of excursions $\times$ the behaviour of the single one that finishes you.

The Keys

1. The object is drawdown, and it has an exponential signature.

Take a Lévy process X with no upward jumps; $Y = \bar{X} - X$ is its distance below its own running peak — drawdown. Ask when Y first exceeds a . The prize: the running peak $\bar{X}$ at that moment is **exponentially distributed with rate $W'(a)/W(a)$** . How high the equity curve climbs before it takes an a -sized hit is memoryless.

2. The mechanism: excursions are a Poisson process, so the problem factorises.

Each dip away from the peak is an "excursion", and Itô's theorem makes them a Poisson point process with measure n . Excursion *heights* obey $n(\text{height} > a) = W'(a)/W(a)$. The Master formula then splits it: survive every ordinary dip up to peak m , then one goes too deep. Those factors multiply, and everything reduces to **scale functions** — the pair already solving two-sided exit.

3. Two ways to die, and only one exists without a Gaussian part.

Theorem 1 covers crossing a **by a jump**; Theorem 2 covers **creeping** smoothly over it, and it is identically zero when $\sigma = 0$. The first theorem returns the full joint law of $(\tau_a, \text{peak}, \text{position just before, jump size})$, extending Avram–Kyprianou–Pistorius, who had only the time and exit position.

4. The embedding works, and the paper names its own two weaknesses.

For any atomless target v , Y stopped at an explicit *moving* boundary on drawdown has law v — no randomisation. But the stopping time is **not minimal**, only weakly so; and if X oscillates and has downward jumps, $E[T^*] = \infty$ — the overshoot costs infinite expected time.

AI's Take

Pure theory. Twenty-one pages, all proofs, no data, no simulation. Half the paper extends a result the author co-wrote, and the other half is superseded by a contemporary he cites himself. What is genuinely new is small and sharp: the joint law including the overshoot and the pre-jump position.

The transferable idea is the factorisation. When something fails after many attempts, do not model the whole trajectory — model the **rate of attempts** and the **conditional damage of the one that lands**. That is the hazard-rate shape, and it is why $W'(a)/W(a)$ is doing all the work here.

One line to keep: a process that only jumps down can never ease across a limit — it always overshoots. Any risk limit assuming you exit *at* the threshold is wrong by the size of a jump.