

Further Analysis: Hidden Markov Models and Market Structure

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AI-Symbiosis Research · April 2026

Overview

Following Lab 4 (Baum HMM Regime Detector on BTC) and the KEGA gap analysis of Cappé-Moulines-Rydén, three results emerged from the intersection of theory and empirical observation. The first is proven. The second is a strong hypothesis. The third is an open conjecture with a testable experiment.

Key Result 1 — The Candle Manifold (Proven)

The observation vector $(\text{fracChange}, \text{fracHigh}, \text{fracLow})$ does not live in flat $\mathbb{R}^3$. It lives on a **constrained submanifold** defined by:

$$\text{fracHigh} \geq 0, \text{fracLow} \geq 0, \text{fracHigh} \geq \text{fracChange}, \text{fracLow} \geq -\text{fracChange}$$

These are hard geometric constraints — not all of $\mathbb{R}^3$ is reachable by a valid candle. The emission space is a bounded, curved surface.

Standard Gaussian HMM fits ellipsoids in flat $\mathbb{R}^3$, ignoring this curvature. The natural gradient upgrade (Baum-Sell manifold paper, Amari 1998) respects the geometry by replacing the flat M-step with:

$$\theta_{t+1} = \theta_t + \eta \cdot G(\theta_t) - \frac{1}{\nabla^2 Q(\theta_t; \theta_t)}$$

where $G(\theta)$ is the Fisher information matrix. This is Gap 2 from the KEGA analysis — the missing bridge between Baum 1970 and information geometry. The parameter space of any probabilistic automaton is already a manifold by construction (product of simplices). The flat EM ignores this. The natural gradient does not.

Implication for Lab 4: The regime boundaries learned by standard Baum-Welch are hyperplanes in flat space. The true regime boundaries are geodesics on the candle manifold. The natural gradient version would find sharper, more stable regime separations.

Key Result 2 — The Automaton-Manifold Correspondence (Strong Hypothesis)

Observing that markets were modeled as a Game Theory Automaton (Fortuna project) and independently as an HMM (Lab 4), and that both arrived at the same architecture, raises a general question:

Does every automaton system have an underlying manifold structure?

What is proven:

- The parameter space of any probabilistic automaton is a Riemannian manifold under the Fisher information metric. This follows directly from Amari (1985) and is not in dispute.
- The HMM is the canonical smooth relaxation of a deterministic finite automaton. Sharp state transitions become probabilistic; the automaton's skeleton becomes a manifold's flesh.

The hypothesis:

Every system with hidden discrete states driving observable outputs has a natural manifold relaxation. The automaton is the discretization. The manifold is the continuous ground truth.

System	Automaton	Manifold Relaxation
Market regimes	Game Theory Automaton	Gaussian HMM on candle manifold
Language	Formal grammar	Transformer attention manifold
Brain states	Neural firing patterns	State-space model on neural manifold
Protein folding	Conformational states	Energy landscape manifold

The direction **automaton** → **manifold** (smooth it) is always well-defined. The reverse direction **manifold** → **automaton** (discretize it) requires choosing a cell decomposition — and that choice is not canonical. This is precisely the model selection problem (Gap 5, KEGA): how many states? The answer depends on the resolution at which you discretize the manifold.

Status: Proven for Markov chains and HMMs. Open for pushdown automata and beyond. Strong enough to publish as a conjecture with the proven cases as supporting evidence.

Key Result 3 — Recursive Sub-Regimes (Conjecture + Testable Experiment)

Lab 4 identifies three macro-regimes: Bull, Bear, Choppy. But within each macro-regime, sub-structure likely exists — a Bull regime contains Strong Bull, Weak Bull, and Topping Bull days. A Bear regime contains Panic Bear, Slow Bear, and Bottoming Bear days.

The conjecture:

Market regimes have recursive structure. Each macro-regime contains distinct micro-regimes with their own emission statistics. The correct model is a Hierarchical HMM (HHMM), not a flat one. This corresponds to a multi-scale decomposition of the candle shape manifold — macro curvature separating Bull from Bear, micro curvature separating sub-states within each.

The experiment (Lab 4b):

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Step 1 – Use Lab 4 labels: Bull / Bear / Choppy per bar  
Step 2 – Extract each subset: df_bull, df_bear, df_choppy  
Step 3 – Fit child HMM(n=3) on each subset independently  
Step 4 – Compare log-likelihood: HHMM vs flat 9-state HMM  
Step 5 – Plot all 9 sub-regimes on the price chart
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Validation criterion: A 3×3 HHMM has ~45 parameters. A flat 9-state HMM has ~99 parameters. If the HHMM matches flat performance at half the parameter count, the hierarchical structure is real and the manifold has genuine multi-scale geometry.

If validated: The answer to "how many states?" is resolution-dependent — approximately 3d states at depth d. The market manifold is fractal-like across scales.

If falsified: The manifold is flat below the macro level — also an important result, constraining the model architecture for production systems.

Summary

Result	Status	Next Step
1. Candle manifold — natural gradient upgrade	Proven (math)	Implement in Lab 4b
2. Automaton-manifold correspondence	Strong hypo-thesis	Prove for input-output HMMs
3. Recursive sub-regimes — HHMM	Open conjecture	Run Lab 4b experiment

These three results form a coherent research arc: the observation space has manifold geometry (Result 1), all automaton systems share this structure (Result 2), and the manifold has recursive sub-structure at multiple scales (Result 3). Lab 4b tests Result 3 empirically while simultaneously providing a testbed for the natural gradient implementation of Result 1.

References

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